

Leptogenesis: The Other Cuts

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Abstract

For standard leptogenesis from the decay of singlet right-handed neutrinos, we derive source terms for the lepton asymmetry that are present in a finite density background but absent in the vacuum. These arise from cuts through the vertex correction to the decay asymmetry, where in the loop either the Higgs boson and the right-handed neutrino or the left-handed lepton and the right-handed neutrino are simultaneously on shell. We evaluate the source terms numerically and use them to calculate the lepton asymmetry for illustrative points in parameter space, where we consider only two right-handed neutrinos for simplicity. Compared to calculations where only the standard cut through the propagators of left-handed lepton and Higgs boson is included, sizable corrections arise when the masses of the right-handed neutrinos are of the same order, but the new sources are found to be most relevant when the decaying right-handed neutrino is heavier than the one in the loop. In that situation, they can yield the dominant contribution to the lepton asymmetry.

1 Introduction

Leptogenesis [1] is often studied in parametric regimes where the masses of the right-handed neutrinos are either hierarchical or degenerate. The hierarchical limit is particularly useful for gaining valuable analytical insights into the connections between leptogenesis and the observed neutrino oscillations [2–4]. To be specific, we discuss here the simple case of two right-handed neutrinos $N_{1,2}$ with masses $M_{1,2}$ and Yukawa couplings $Y_{1,2}$ to the Higgs and lepton doublets, *i.e.* a model as specified in Ref. [5]. The key simplification in the hierarchical limit, $M_1 \ll M_2$, is that the evolution of the lepton asymmetry during leptogenesis as a function of $z = M_1/T$ depends up to a proportionality factor only on the ratio $M_1/(Y_1^2 m_{\text{Pl}})$, which characterises the washout strength. Here, T is the temperature and m_{Pl} is the Planck mass. The remaining proportionality factor characterising the amount of CP violation is $\text{Im}[Y_1^2 Y_2^{*2}] M_1/M_2$, up to corrections of order M_1^3/M_2^3 , which one neglects in the hierarchical limit. On the other hand, mass-degenerate right-handed neutrinos lead to resonant leptogenesis [6–10]. This corresponds to a phenomenologically attractive scenario, since the decay asymmetry is enhanced, which allows for lower temperatures at which leptogenesis takes place. Thus, the production of unwanted relics, most notoriously of gravitinos within supersymmetric models [11–13], along with the lepton asymmetry asymmetry can be avoided. A lower energy scale might also give rise to new experimentally accessible signals connected with leptogenesis, see *e.g.* [14–16].

However, the origin of the masses of the right-handed neutrinos is yet unknown and their masses may well be neither hierarchical nor degenerate. When the hierarchical limit $M_1 \ll M_2$ no longer applies, it is well known that the decay asymmetry of N_1 is not simply proportional to M_1/M_2 with negligible corrections, as can be verified by inspection of the vertex and wave-function contributions to the decay asymmetry of N_1 [1, 9]. In the finite-temperature background, there are additional corrections due to new cuts. While in the vacuum background, the CP -violating contribution from the vertex function arises exclusively from the cut through $\{\ell, \phi\}$, where the internal lepton and Higgs boson are on-shell, at finite temperature also the two other possible cuts through $\{\ell, N_2\}$ or $\{\phi, N_2\}$ in the vertex diagram contribute, *cf.* Figure 1 (B). This is because in the finite-temperature background, the cut-particles do not need to correspond to stimulated (suppressed) emission processes for bosons (fermions), but they can also correspond to absorption processes of particles from the plasma. The presence of these cut contributions has been mentioned in Ref. [17]. However, by now, neither analytical expressions for these terms have been derived nor have these been evaluated numerically in order to compute effective decay asymmetries or the resulting lepton asymmetry. These are the main goals of the present work.

The new cut contributions are a finite density effect, and a powerful method of describing out-of-equilibrium field theory is given by the Schwinger-Keldysh Closed-Time-Path (CTP) formalism [18, 19]. This approach has been applied to leptogenesis and has resulted in some recent activity which we build upon within the present work [20–27]. Main advantages of the CTP approach to leptogenesis over the conventional description

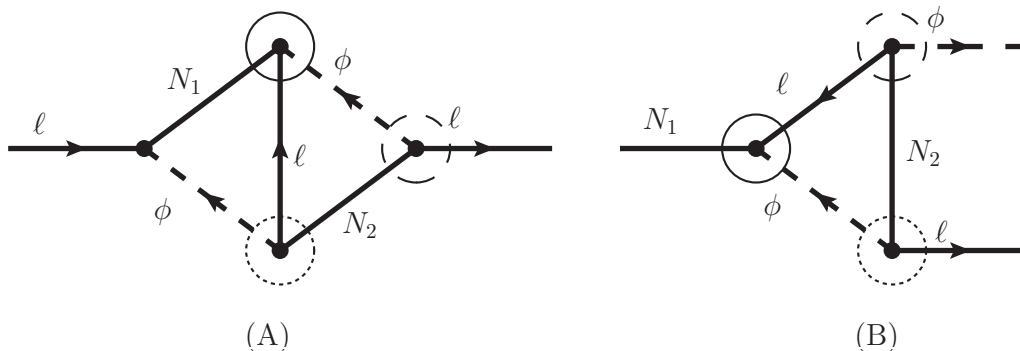


Figure 1: Diagram (A) represents the vertex correction $\Sigma_\ell^{v>}$ to the lepton self-energy in the CTP formalism. Diagram (B) is the subdiagram of (A) that accounts for the decays and inverse decays of the out-of-equilibrium particle N_1 . We indicate the various cuts that arise from demanding that the cut particles in the loop are on shell. The solid circle represents the standard cut through $\{\ell, \phi\}$ that is the only contribution in the vacuum or when finite density effects are neglected. The dashed cut is the contribution for off-shell ℓ through $\{\phi, N_2\}$, the dotted cut the the contribution for off-shell ϕ through $\{\ell, N_2\}$.

by semi-classical Boltzmann equations may be seen in the absence of the need of an explicit subtraction procedure for real intermediate states (RIS) [28] and in the systematic treatment of finite-density corrections.

Within the CTP approach, the vertex diagram in Figure 1 (B) appears as a subdiagram in the self-energy Figure 1 (A), which is a contribution to the lepton self-energy. This self-energy in turn enters the collision term of the Kadanoff-Baym equation for the lepton, that can be reduced to a kinetic equation which describes the gain and the loss and therefore the time evolution of the lepton number density. In order to simplify the collision term to a manageable form, it is useful to substitute equilibrium propagators for ℓ and ϕ and to employ Kubo-Martin-Schwinger (KMS) relations. The present work relies strongly on Ref. [5], where these simplification strategies are explained and justified in detail and where also many definitions and quantities that we use here are introduced. An additional simplifying assumption that we introduce here, but that will not hold true in general, is that also N_2 is maintained in equilibrium (*e.g.* through interactions with an additional lepton flavour), such that N_1 is the only out-of-equilibrium particle. We leave a study of the situation where more than one of the right-handed neutrinos deviates from equilibrium to future work.

Provided the initial distribution function for N_1 is thermal, as we assume in the present work, the main contributions to the lepton asymmetry occur at times when the value of $z = M_1/T$ is in the range of around one up to a few, for an in-detail discussion, see Ref. [29]. This is because at these temperatures, N_1 becomes non-relativistic and therefore deviate from equilibrium. As a consequence, we anticipate the new cuts to be important under the following conditions:

- Since the effect is due to the finite densities, M_2 should not be much larger than M_1 , because otherwise the new contributions are Maxwell-suppressed, just as the equilibrium distribution functions for energies of order M_2 much larger than T .
- The new effects will be most pronounced in case leptogenesis occurs at comparably low values for z , when it is around one. This is the case in the transitional regime between strong and weak washout. In such a situation, the finite density effects that include the contributions from the new cuts, take their largest relevance.
- A loophole to these arguments are situations where $M_1 > M_2$. (We denote within this work by N_1 the neutrino that deviates from equilibrium, whereas N_2 is assumed to be very close to equilibrium at the time relevant for leptogenesis. This definition differs from what is commonly used in the literature, where N_1 corresponds to the lightest right-handed neutrino, *cf.* Refs [30–32]. In this case, the finite density effects from the N_2 are also important when leptogenesis occurs at larger values of z , since the distribution of N_2 can still be unsuppressed.

The plan of this paper is as follows: In Section 2, we give the expressions for the CP -violating source terms that bias the lepton number and that are valid for finite ratios of M_1/M_2 . For the standard cut through $\{\ell, \phi\}$, we quote the result from Ref. [5], while for the new cuts through $\{\ell, N_2\}$ and $\{\phi, N_2\}$, respectively, we derive new expressions. In Section 3, we present the first numerical results for the cut through $\{\ell, \phi\}$ at finite M_1/M_2 and finite density, as well as for the new cuts through $\{\ell, N_2\}$ and $\{\phi, N_2\}$. We first define and evaluate expressions for the effective CP violation for several ratios of M_1/M_2 . Then, we solve the kinetic equations for the lepton asymmetry for the same values of M_1/M_2 and specific illustrative choices of Y_1, Y_2 . The results for the lepton asymmetry are compared with the effective CP violation as well as with what is stated above on the parametric regions where we anticipate the new cut contributions to be most relevant. Within flavoured leptogenesis, a new cut in the wave-function of the lepton ℓ contributes to the asymmetry. A rough estimate of this effect is presented in Section 4, and it is found to be negligibly small. We summarise and conclude in Section 5.

2 Thermal Vertex Function

2.1 Vertex Self Energy and Collision Term

Within the present work, we extend the results of Ref. [5]. There, the kinetic evolution equation for the lepton asymmetry is expressed as

$$\frac{d}{d\eta} (n_\ell - \bar{n}_\ell) = W + S, \quad (1)$$

where η is the conformal time in the Friedmann background, W the washout term, S the source term and n_ℓ (\bar{n}_ℓ) the comoving (anti-)lepton number density. In the radiation-dominated Universe, the scale factor is given by $a = a_R \eta$, where a_R is an arbitrary

constant. The washout term W is discussed in detail in Ref. [5]. It encompasses the tree-level decay and inverse decay processes of the right-handed neutrino. The source term accounts for the CP -violating loop effects. It decomposes as

$$S = \int \frac{d^3k}{(2\pi)^3} [\mathcal{C}_\ell^{\text{wf}}(\mathbf{k}) + \mathcal{C}_\ell^{\text{v}}(\mathbf{k})] , \quad (2)$$

where $\mathcal{C}_\ell^{\text{wf}}$ is the wave-function contribution to the collision term, as it is given in Ref. [5]. In this work, we are primarily concerned with the vertex contribution $\mathcal{C}_\ell^{\text{v}}$ and the new corrections that it acquires when compared with Ref. [5]. This term can be expressed in the usual Kadanoff-Baym form

$$\mathcal{C}_\ell^X(\mathbf{k}) = \int \frac{dk^0}{2\pi} \text{tr} \left[i\mathbb{Z}_\ell^{X>}(k) P_L iS_\ell^{<}(k) - i\mathbb{Z}_\ell^{X<}(k) P_L iS_\ell^{>}(k) \right] , \quad (3)$$

for $X \equiv \text{v}$. The Wightman self-energy $\mathbb{Z}_\ell^{\text{v}>}$ has the diagrammatic representation of Figure 1 (A). It is given by [5]

$$i\mathbb{Z}_\ell^{\text{v}>}(k) = -Y_i^{*2} Y_j^2 \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \quad (4)$$

$$\begin{aligned} & \left\{ iS_{Ni}^{>}(-p) C [iS_\ell^T(p+k+q)]^t C^\dagger iS_{Nj}^T(-q) i\Delta_\phi^{<}(-p-k) i\Delta_\phi^T(-q-k) \right. \\ & - iS_{Ni}^{\bar{T}}(-p) C [iS_\ell^{<}(p+k+q)]^t C^\dagger iS_{Nj}^T(-q) i\Delta_\phi^{<}(-p-k) i\Delta_\phi^{<}(-q-k) \\ & - iS_{Ni}^{>}(-p) C [iS_\ell^{>}(p+k+q)]^t C^\dagger iS_{Nj}^{>}(-q) i\Delta_\phi^{\bar{T}}(-p-k) i\Delta_\phi^T(-q-k) \\ & \left. + iS_{Ni}^{\bar{T}}(-p) C [iS_\ell^{\bar{T}}(p+k+q)]^t C^\dagger iS_{Nj}^{>}(-q) i\Delta_\phi^{\bar{T}}(-p-k) i\Delta_\phi^{<}(-q-k) \right\} , \end{aligned}$$

where we sum over the indices i, j . The self-energy $\mathbb{Z}_\ell^{\text{v}<}$ follows when applying the replacements $<\leftrightarrow>$ and $T \leftrightarrow \bar{T}$. For the propagators S_ℓ , Δ_ϕ and S_{Ni} , we use the zero-width approximations as written down in Ref. [5].

To be specific, let us now consider the term with $i = 1$ and $j = 2$. The CP -violating contributions from the decays and inverse decays of N_1 arise when two out of the three propagators $iS_\ell(p+k+q)$, $i\Delta_\phi(-p-k)$ and $iS_{N2}(-q)$ are on shell. These are the cut propagators. As a consequence of this, the third propagator is off shell. Since the $>$ and $<$ propagators are purely on shell, it follows that only terms where the off-shell propagator is of the time-ordered T or anti-time-ordered \bar{T} type contribute. The collision term (3) can therefore be split into the portions

$$\mathcal{C}_\ell^{\text{v}} = \mathcal{C}_\ell^{\text{v}\ell\phi} + \mathcal{C}_\ell^{\text{v}\phi N2} + \mathcal{C}_\ell^{\text{v}\ell N2} , \quad (5)$$

where the superscripts indicate through which two of the loop propagators in the vertex correction the cut goes. Likewise, we decompose the vertex contribution to the source term as

$$S^{\text{v}} = S^{\text{v}\ell\phi} + S^{\text{v}\phi N2} + S^{\text{v}\ell N2} . \quad (6)$$

Within the present work, we make the simplifying assumption that N_2 is in thermal equilibrium at all times, whereas N_1 is in equilibrium initially but then deviates from equilibrium when it becomes non-relativistic. We note that in general, both N_1 and N_2 will deviate from equilibrium at the same time. In fact, situations are conceivable where the initial abundance of N_2 is zero and Y_2 is so small that N_2 does not equilibrate before becoming non-relativistic. Therefore, the equilibrium deviation of N_2 can be large when compared to N_1 , and inverse decays of N_2 may largely enhance the lepton asymmetry. For now, leave this interesting possibility for future work and note that N_2 may be maintained in equilibrium by a stronger coupling to a different lepton flavour within which no asymmetry is produced.

2.2 Cut through $\{\ell, \phi\}$

The term $\mathcal{C}_\ell^{\nu\ell\phi}$ arises from those terms within $i\Sigma_\ell^{v<, >}$, Eq. (4), where the propagators $iS_{N_2}^{T, \bar{T}}$ occur. It can be further simplified when approximating the distribution functions of ℓ and ϕ by Fermi-Dirac and, respectively, Bose-Einstein equilibrium distributions $f_\ell^{\text{eq}}(\mathbf{k})$ and $f_\phi^{\text{eq}}(\mathbf{k})$. Accounting for the fact that ℓ deviates from equilibrium only leads to higher order corrections in the gradient expansion, as it is explained in Ref. [5]. The gradient expansion corresponds to an expansion in powers of Y_1^2 or equivalently H/T , where H denotes the Hubble rate.

The contribution of the cut through $\{\ell, \phi\}$ to the source term can be factorised into [5]

$$\begin{aligned} \Gamma_\mu^{\ell\phi}(k, p''; M_1, M_2) &= \int \frac{d^3 k'}{(2\pi)^3 2|\mathbf{k}'|} \frac{d^3 k''}{(2\pi)^3 2|\mathbf{k}''|} (2\pi)^4 \delta^4(k - k' - k'') k'_\mu \frac{M_1 M_2}{(k' - p'')^2 - M_2^2} \\ &\quad \times [1 - f_\ell^{\text{eq}}(\mathbf{k}') + f_\phi^{\text{eq}}(\mathbf{k}'')] \end{aligned} \quad (7)$$

and

$$\begin{aligned} V^{\ell\phi}(k, M_1, M_2) &= \int \frac{d^3 p'}{(2\pi)^3 2|\mathbf{p}'|} \frac{d^3 p''}{(2\pi)^3 2|\mathbf{p}''|} (2\pi)^4 \delta^4(k - p' - p'') p'^\mu \Gamma_\mu^{\ell\phi}(k, p''; M_1, M_2) \\ &\quad \times [1 - f_\ell^{\text{eq}}(\mathbf{p}') + f_\phi^{\text{eq}}(\mathbf{p}'')] \end{aligned} \quad (8)$$

with the result

$$S^{\nu\ell\phi} = \int \frac{d^3 p'}{(2\pi)^3} \mathcal{C}_\ell^{\nu\ell\phi}(\mathbf{p}') = 4 \text{Im}[Y_1^2 Y_2^{*2}] \int \frac{d^3 k}{(2\pi)^3 2\sqrt{\mathbf{k}^2 + M_1^2}} \delta f_{N_1}(\mathbf{k}) V^{\ell\phi}(k; M_1, M_2). \quad (9)$$

The term $\delta f_{N_1}(\mathbf{k})$ denotes the deviation of $f_{N_1}(\mathbf{k})$ from the equilibrium Fermi-Dirac distribution, $\delta f_{N_1}(\mathbf{k}) = f_{N_1}(\mathbf{k}) - f_{N_1}^{\text{eq}}(\mathbf{k})$.

In Ref. [5], this result has been evaluated in the hierarchical limit $M_1/M_2 \ll 1$, which has the virtue that, as it is explained in the Introduction, the evolution of the lepton number density is independent of M_2 up to an overall proportionality. Furthermore, when

$M_1/M_2 \ll 1$, the collision term (9) can be reduced analytically to a one-dimensional integral, which can easily be evaluated numerically. For the present work, we numerically evaluate a multi-dimensional integral for $S^{\nu\ell\phi}$ that remains when exploiting the δ -functions. We note that also within the related articles [22, 24], all source terms have been evaluated in the hierarchical limit, such that here, we present the first quantitative results for effects of finite M_1/M_2 in a finite-density background.

2.3 Cut through $\{\phi, N_2\}$

Now the cut goes through the internal N_2 and ϕ lines of Figure 1 (A), and the internal ℓ is off-shell. The according contributions to the collision term are

$$\begin{aligned} \mathcal{C}_\ell^{\nu\phi N_2}(\mathbf{k}) = & -Y_i^{*2}Y_j^2 \int \frac{dk^0}{2\pi} \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \\ & \text{tr} \left\{ iS_\ell^<(k) iS_{N_i}^>(-p) C [S_\ell^T(p+k+q)]^t C^\dagger iS_{N_j}^T(-q) i\Delta_\phi^<(-p-k) i\Delta_\phi^T(-q-k) \right. \\ & - iS_\ell^<(k) iS_{N_i}^{\bar{T}}(-p) C [S_\ell^T(p+k+q)]^t C^\dagger iS_{N_j}^>(-q) i\Delta_\phi^{\bar{T}}(-p-k) i\Delta_\phi^<(-q-k) \\ & + iS_\ell^>(k) iS_{N_i}^<(-p) C [S_\ell^T(p+k+q)]^t C^\dagger iS_{N_j}^{\bar{T}}(-q) i\Delta_\phi^>(-p-k) i\Delta_\phi^{\bar{T}}(-q-k) \\ & \left. - iS_\ell^>(k) iS_{N_i}^T(-p) C [S_\ell^T(p+k+q)]^t C^\dagger iS_{N_j}^<(-q) i\Delta_\phi^T(-p-k) i\Delta_\phi^>(-q-k) \right\}. \end{aligned} \quad (10)$$

As explained above, we assume here that besides ℓ and ϕ , N_2 is in equilibrium, such that its number density is given by the Fermi-Dirac distribution function $f_{N_2}^{\text{eq}}(\mathbf{k})$. This allows us to apply KMS relations and two further replacements, which become identities under the integrals (*cf.* Ref. [5], where this is explained in more detail):

$$\begin{aligned} iS_{N_i}^{T,\bar{T}}(-p) i\Delta_\phi^{T\bar{T}}(-p-k) & \rightarrow \frac{1}{2} [iS_{N_i}^<(-p) i\Delta_\phi^>(-p-k) + iS_{N_i}^>(-p) i\Delta_\phi^<(-p-k)] , \\ iS_{N_j}^{T,\bar{T}}(-q) i\Delta_\phi^{T,\bar{T}}(-q-k) & \rightarrow \frac{1}{2} [iS_{N_j}^<(-q) i\Delta_\phi^>(-q-k) + iS_{N_j}^>(-q) i\Delta_\phi^<(-q-k)] . \end{aligned} \quad (11)$$

The result of these simplifications is

$$\begin{aligned} \mathcal{C}_\ell^{\nu\phi N_2}(\mathbf{k}) = & -[Y_1^{*2}Y_2^2 - Y_1^2Y_2^{*2}] \int \frac{dk^0}{2\pi} \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \\ & \frac{1}{2} \text{tr} \left\{ [iS_\ell^<(k) iS_{N_1}^>(-p) C [S_\ell^T(p+k+q)]^t C^\dagger i\Delta_\phi^<(-p-k) \right. \\ & \quad - iS_\ell^>(k) iS_{N_1}^<(-p) C [S_\ell^T(p+k+q)]^t C^\dagger i\Delta_\phi^>(-p-k)] \\ & \quad \times [iS_{N_2}^<(-q) i\Delta_\phi^>(-q-k) - iS_{N_2}^>(-q) i\Delta_\phi^<(-q-k)] \left. \right\} . \end{aligned} \quad (12)$$

The final contribution to the source term,

$$S^{\nu\phi N_2} = \int \frac{d^3 p'}{(2\pi)^3} \mathcal{C}_\ell^{\nu\phi N_2}(\mathbf{p}'), \quad (13)$$

then follows when substituting the explicit forms of the propagators (as they can be found in Ref. [5]) as

$$\begin{aligned} S^{\nu\phi N_2} = & 4\text{Im}[Y_1^2 Y_2^{*2}] \int \frac{d^3 p'}{(2\pi)^3 2|\mathbf{p}'|} \frac{d^3 p}{(2\pi)^3 2\sqrt{\mathbf{p}^2 + M_1^2}} \frac{d^3 p''}{(2\pi)^3 |\mathbf{p}''|} \frac{d^3 k''}{(2\pi)^3 2|\mathbf{k}''|} \frac{d^3 k'}{(2\pi)^3 2\sqrt{\mathbf{k}'^2 + M_2^2}} \\ & \times (2\pi)^4 \delta^4(p - p' - p'') (2\pi)^4 \delta^4(k' - k'' - p') p'^\mu \frac{(p + k'')_\mu}{(p + k'')^2} M_1 M_2 \\ & \times \delta f_{N_1}(\mathbf{p}) [1 - f_\ell^{\text{eq}}(\mathbf{p}') + f_\phi^{\text{eq}}(\mathbf{p}'')] \times [-f_\phi^{\text{eq}}(\mathbf{k}'') - f_{N_2}^{\text{eq}}(\mathbf{k}')] . \end{aligned} \quad (14)$$

To our knowledge, this is the first report of a result for a source term for leptogenesis that is present at finite density but absent in the limit of a vacuum background. We remark that the last factor results from the expression $-f_\phi^{\text{eq}} - f_{N_2}^{\text{eq}} = -[1 + f_\phi^{\text{eq}}]f_{N_2}^{\text{eq}} - f_\phi^{\text{eq}}[1 - f_{N_2}^{\text{eq}}]$. This could also be guessed starting from a hypothetical loop factor when the decay $\ell \rightarrow \phi N_2$ was kinematically allowed, $[1 + f_\phi^{\text{eq}} - f_{N_2}^{\text{eq}}] = [1 + f_\phi^{\text{eq}}][1 - f_{N_2}^{\text{eq}}] + f_\phi^{\text{eq}} f_{N_2}^{\text{eq}}$, and applying the replacements $-f_{N_2}^{\text{eq}} \leftrightarrow [1 - f_{N_2}^{\text{eq}}]$. This argument may be considered as a consistency check for our derived result (14).

It is important to notice that for $M_2 \gg T$, the source term (14) is strongly Maxwell suppressed. While in such a situation, $f_{N_2}^{\text{eq}}(\mathbf{k}')$ is always suppressed because of the large mass of N_2 , the energy-momentum conserving δ -functions always imply that then at least one of the momenta \mathbf{p} or \mathbf{k}'' is much larger than T , such that $f_\phi^{\text{eq}}(\mathbf{k}'')$ or $\delta f_{N_1}(\mathbf{p})$ are suppressed as well.

2.4 Cut through $\{\ell, N_2\}$

We finally consider the cuts through the propagators N_2 and ℓ within the loop and take ϕ to be off-shell. The contribution to the collision term is

$$\begin{aligned} \mathcal{C}_\ell^{\nu\ell N_2}(\mathbf{k}) = & -Y_i^{*2} Y_j^2 \int \frac{dk^0}{2\pi} \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \\ & \text{tr} \left\{ iS_\ell^<(k) iS_{N_i}^>(-p) C [S_\ell^>(p + k + q)]^t C^\dagger iS_{N_j}^>(-q) i\Delta_\phi^T(-p - k) i\Delta_\phi^T(-q - k) \right. \\ & - iS_\ell^<(k) iS_{N_i}^{\bar{T}}(-p) C [S_\ell^{\bar{T}}(p + k + q)]^t C^\dagger iS_{N_j}^>(-q) i\Delta_\phi^T(-p - k) i\Delta_\phi^<(-q - k) \\ & + iS_\ell^>(k) iS_{N_i}^<(-p) C [S_\ell^<(p + k + q)]^t C^\dagger iS_{N_j}^<(-q) i\Delta_\phi^T(-p - k) i\Delta_\phi^{\bar{T}}(-q - k) \\ & \left. - iS_\ell^>(k) iS_{N_i}^T(-p) C [S_\ell^T(p + k + q)]^t C^\dagger iS_{N_j}^<(-q) i\Delta_\phi^T(-p - k) i\Delta_\phi^>(-q - k) \right\} . \end{aligned} \quad (15)$$

Again, we substitute for S_{N_2} , S_ℓ and S_ϕ the equilibrium propagators and employ KMS relations. Under the integrals, we make the replacements

$$\begin{aligned} & iS_{N_i}^{T,\bar{T}}(-p) iC \left[S_\ell^{T,\bar{T}}(p+k+q) \right]^t C^\dagger \\ & \rightarrow \frac{1}{2} \left[iS_{N_i}^>(-p) iC \left[S_\ell^>(p+k+q) \right]^t C^\dagger + iS_{N_i}^<(-p) iC \left[S_\ell^<(p+k+q) \right]^t C^\dagger \right]. \end{aligned} \quad (16)$$

Furthermore, it is useful to notice that when substituting the equilibrium propagator for ℓ ,

$$\begin{aligned} & iS_\ell^<(k) iS_{N_i}^>(-p) C \left[S_\ell^>(p+k+q) \right]^t C^\dagger iS_{N_j}^>(-q) \\ & - iS_\ell^>(k) iS_{N_i}^<(-p) C \left[S_\ell^<(p+k+q) \right]^t C^\dagger iS_{N_j}^<(-q) \end{aligned}$$

is odd under the exchange $k^0, p^0, q^0 \rightarrow -k^0, -p^0, -q^0$, while $\text{Im}[i\Delta_\phi^{T,\bar{T}}(-q-k)]$ (which is the off-shell contribution) is even. Making use of these additional remarks, the collision term simplifies to

$$\begin{aligned} \mathcal{C}_\ell^{\nu\ell N_2} = & -[Y_1^{*2} Y_2^2 - Y_1^2 Y_2^{*2}] \int \frac{dk^0}{2\pi} \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \\ & \text{tr} \left\{ [iS_{N_2}^<(-q) iS_\ell^>(k) - iS_{N_2}^>(-q) iS_\ell^<(k)] i\Delta_\phi^T(-p-k) \right. \\ & \times [iS_{N_1}^<(-p) C \left[S_\ell^<(p+k+q) \right]^t C^\dagger i\Delta_\phi^<(-q-k) \\ & \left. - iS_{N_1}^>(-p) C \left[S_\ell^>(p+k+q) \right]^t C^\dagger i\Delta_\phi^>(-q-k)] \right\}. \end{aligned} \quad (17)$$

The source term is

$$S^{\nu\ell N_2} = \int \frac{d^3 p'}{(2\pi)^3} \mathcal{C}_\phi^{\nu\ell N_2}(\mathbf{p}'), \quad (18)$$

which becomes, when substituting the finite-density propagators,

$$\begin{aligned} S^{\nu\ell N_2} = & 4\text{Im}[Y_1^2 Y_2^{*2}] \int \frac{d^3 k''}{(2\pi)^3 2|\mathbf{k}''|} \frac{d^3 p}{(2\pi)^3 2\sqrt{\mathbf{p}^2 + M_1^2}} \frac{d^3 p''}{(2\pi)^3 2|\mathbf{p}''|} \frac{d^3 k'}{(2\pi)^3 2\sqrt{\mathbf{k}'^2 + M_2^2}} \frac{d^3 p'}{(2\pi)^3 2|\mathbf{p}'|} \\ & \times (2\pi)^4 \delta^4(p - k'' - p'') (2\pi)^4 \delta^4(k' - k'' - p') \frac{p''_\mu p'^\mu}{(p'' + k')^2} M_1 M_2 \\ & \times \delta f_{N_1}(\mathbf{p}) [1 - f_\ell^{\text{eq}}(\mathbf{p}'') + f_\phi^{\text{eq}}(\mathbf{k}'')] \times [f_\ell^{\text{eq}}(\mathbf{p}') - f_{N_2}^{\text{eq}}(\mathbf{k}')] . \end{aligned} \quad (19)$$

Again, we note that the factor $[f_\ell^{\text{eq}} - f_{N_2}^{\text{eq}}] = -[1 - f_\ell^{\text{eq}}] f_{N_2}^{\text{eq}} + f_\ell^{\text{eq}} [1 - f_{N_2}^{\text{eq}}]$ could also be guessed from a would-be factor $[1 - f_\ell^{\text{eq}}][1 - f_{N_2}^{\text{eq}}] - f_\ell^{\text{eq}} f_{N_2}^{\text{eq}} = 1 - f_\ell^{\text{eq}} f_{N_2}^{\text{eq}}$, which would arise if $\phi \rightarrow \ell N_2$ was kinematically allowed, and the replacements $-f_{N_2}^{\text{eq}} \leftrightarrow [1 - f_{N_2}^{\text{eq}}]$. We also note that $S^{\nu\ell N_2}$ is Maxwell suppressed for $M_2 \gg T$, in analogy with what is discussed $S^{\nu\phi N_2}$.

3 Examples

We now present results from the numerical evaluation of the source terms (9,14,19). In Section 3.1, we define an effective measure for the CP violation from the various cut contributions through a benchmark out-of-equilibrium distribution for N_1 . Then, we evaluate this effective CP violation as a function of $z = M_1/T$ for various values of M_1/M_2 . We find that in case $M_2 > M_1$ the new corrections are only significant for $z \sim 1$ or smaller. If $M_2 < M_1$, the corrections from the new cuts can be relevant for larger values of z . We proceed in Section 3.2 with the calculation of the lepton asymmetry by solving the kinetic Boltzmann-type equations (1) with the new source terms. Our choice of the washout strength is motivated by the wish to exhibit models where the contributions from the new cuts are sizable, that is where relevant contributions to the final asymmetry arise at values for z around one. This is the case in the transitional regime from weak to strong washout. We find that indeed, the new cut contributions can have a sizable impact on the final asymmetry. However, in case sizable contributions to the asymmetry originate from $z \ll 1$ in our simulations, the quantitative results need to be interpreted with care. This is because in these regions, thermal corrections to the masses and widths of ϕ and ℓ will become relevant.

3.1 Effective CP -Violating Parameter

In order to obtain a quantitative comparison of the amount of CP violation from the various source terms, it is useful to define a benchmark form for the distribution $\delta f_{N_1}(\mathbf{k})$. This is necessary, since the precise form of $\delta f_{N_1}(\mathbf{k})$ depends on time, washout strength and initial conditions. We follow Ref. [5] by taking for $f_{N_1}(\mathbf{k})$ a Fermi-Dirac distribution with a pseudo-chemical potential μ_{N_1} . The deviation from equilibrium is then obtained by expanding to linear order in μ_{N_1}/T ,

$$\delta f_{N_1}(\mathbf{k}) = f_{N_1}^{\text{eq}}(\mathbf{k}) (1 - f_{N_1}^{\text{eq}}(\mathbf{k})) \frac{\mu_{N_1}}{T}. \quad (20)$$

When the only interactions of the neutrino N_1 are mediated by Y_1 , the actual distribution function is not exactly described by the pseudo-chemical potential, and Eq. (20) should indeed only be regarded as a useful benchmark for the purpose of comparing the various contributions to the source term. Note however that in case there are fast elastic scatterings between the neutrinos N_1 , Eq. (20) should be a very accurate description for the actual distribution. We substitute Eq. (20) into Eqs. (9,14,19) and into S^{wf} and $S_{M_2 \gg M_1}$ as given in Ref. [5], where $S_{M_2 \gg M_1}$ is the source term including both, vertex and wave function terms, evaluated in the hierarchical limit. Out of these, we take the ratios

$$(S^{\text{v}\ell\phi} + S^{\text{wf}})/S_{M_2 \gg M_1} \quad (21)$$

and

$$(S^{\text{v}} + S^{\text{wf}})/S_{M_2 \gg M_1}, \quad (22)$$

as functions of $z = M_1/T$ and with S^ν defined in Eq. (6). The ratio (21) allows for a comparison of the source term at finite density and the standard cut through ℓ and ϕ only with the source term in the hierarchical limit. Through the ratio (22), we compare the source with all cuts at finite density with the hierarchical limit. To quantify the effect of the new cuts, both ratios (21) and (22) are compared with one another.

Before we present the numerical results for Eqs. (21) and (22), a few remarks on their relevance and range of validity are in order. First, note that the relevant contributions to leptogenesis are generated when $z \sim 1$ or larger. (This is to be understood in the sense that while z is *much* smaller than one, no sizable contributions to the final asymmetry are generated). This is a consequence of the fact that N_1 must become non-relativistic and Maxwell suppressed before it equilibrates, because otherwise the lepton asymmetry would be completely washed out. Therefore, the effect of the new contributions is more relevant if it extends to larger values of z .

Second, we remark that for small z , finite temperature effects should become relevant, which we do not take into account within the present work. The most important correction is the contribution of the top quarks to the Debye mass-square of the Higgs boson, $m_H^2 = (1/4)h_t^2 T^2$, where h_t is the top-quark Yukawa coupling. Besides, also the $SU(2)_L$ gauge couplings of the Higgs bosons and the leptons are of relevance. A full evaluation of these finite-temperature effects in the context of leptogenesis has not been performed yet, but it would be of great importance for obtaining quantitatively accurate results in models where asymmetries generated at values of z that are somewhat smaller than one are relevant. Moreover, the size of the top-quark Yukawa coupling at the energy scale of leptogenesis depends due to running on the mass of the Higgs boson, which is yet unknown. For the present discussion, we therefore keep in mind that for the contributions that are generated at $z < 1$, we have to anticipate an inaccuracy of order one.

In Figure 2, the ratios (21) and (22) are plotted for several values of M_1/M_2 . The main features for each of the particular values can be summarised as follows:

- (A): For $M_2/M_1 = 1.1$, the hierarchical limit is of course not a good approximation. The total asymmetry is dominated by the resonantly enhanced contribution S^{wf} . As a consequence of this, $(S^{\nu\ell\phi} + S^{\text{wf}})/S_{M_2 \gg M_1}$ is much larger than one. Since the distribution of N_2 is only very weakly thermally suppressed when compared to N_1 , $S^{\nu\ell\phi} + S^{\text{wf}}$ and $S^\nu + S^{\text{wf}}$ agree only for comparably large values of z .
- (B): For $M_2/M_1 = 2.0$, we note that the hierarchical limit becomes a better approximation to $S^{\nu\ell\phi} + S^{\text{wf}}$, while yet deviating by about 20%. Compared to case (A), $S^{\nu\ell\phi} + S^{\text{wf}}$ and $S^\nu + S^{\text{wf}}$ start to agree for smaller values of z , as the distribution of N_2 now suffers from stronger thermal suppression. We note that the very substantial deviation of $S^{\nu\ell\phi} + S^{\text{wf}}$ from $S^\nu + S^{\text{wf}}$ for very small values of z needs to be interpreted with care in the light of the thermal corrections mentioned above.
- (C): For $M_2/M_1 = 5.0$, the results obtained from the hierarchical limit $S_{M_2 \gg M_1}$ and the source $S^{\nu\ell\phi} + S^{\text{wf}}$ with the standard cuts only are now in good agreement, as

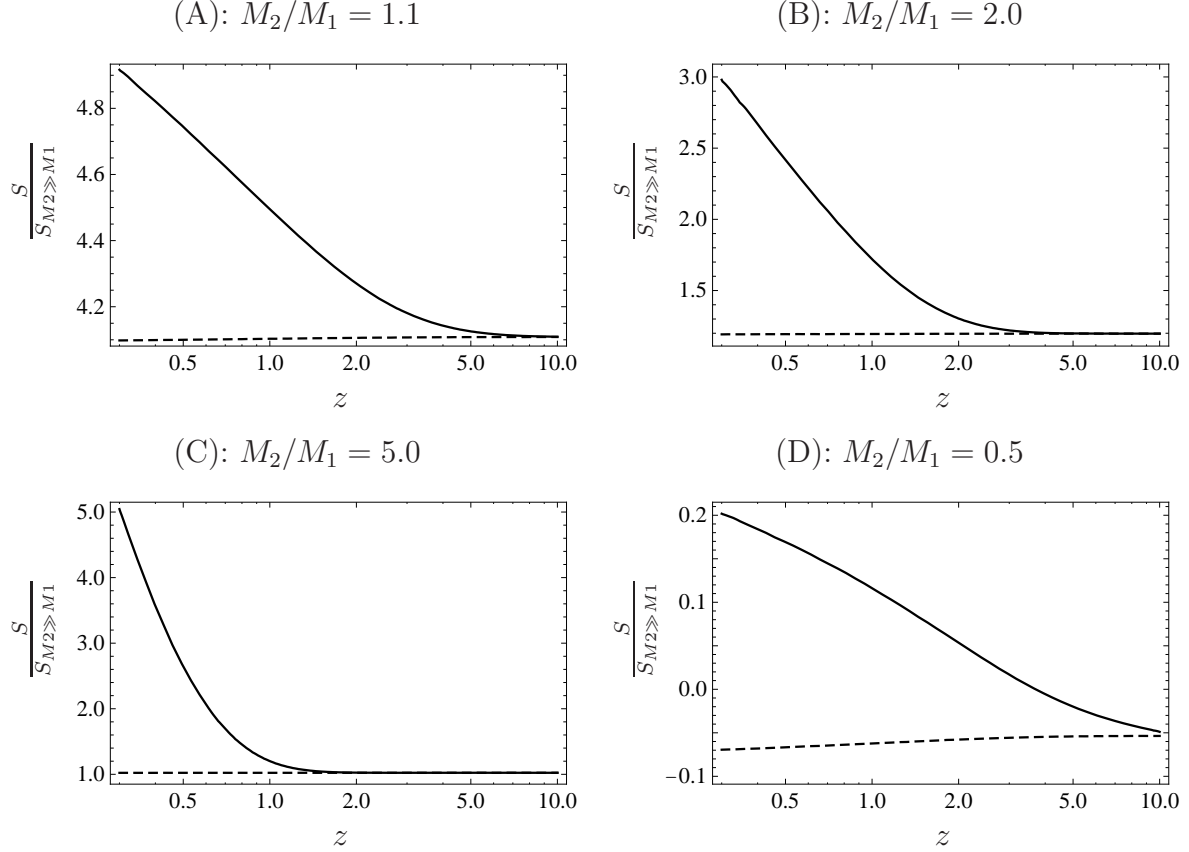


Figure 2: $(S^v + S^{wf})/S_{M2 \gg M1}$ (solid line) and $(S^{v\ell\phi} + S^{wf})/S_{M2 \gg M1}$ (dashed line) over $z = M_1/T$.

anticipated. This serves also as a consistency check for our numerical evaluation of $S^v + S^{wf}$ in the form of a multi-dimensional integral as compared to $S_{M2 \gg M1}$ through the one-dimensional integral given in Ref. [5]. The new cut contributions become suppressed for even smaller values of z along with the stronger Maxwell suppression of N_2 . The comment regarding Scenario (B), that the deviations that originate for values $z \ll 1$ need to be interpreted with care due to thermal corrections, is even more relevant for the present case.

- (D): In this Scenario, $M_2/M_1 = 0.5$. Since N_2 is now lighter than N_1 , the coupling Y_2 must be chosen sufficiently small, such that N_2 does not equilibrate and wash out the lepton asymmetry before it becomes non-relativistic and Maxwell suppressed. In this situation, for the whole relevant range of z , neither the hierarchical limit nor the sources $S^{v\ell\phi} + S^{wf}$ are an accurate approximation to the full result $S^v + S^{wf}$.

3.2 Lepton Asymmetries

From the comparison of the effective amount of CP violation in presence of the new cuts with the case when the new cuts are neglected, we see that when $M_2 > M_1$, the deviations are only sizable for z is of order a few or smaller. For larger values of z , the distribution of the N_2 is strongly Maxwell suppressed, and the new cut contributions become irrelevant. In order to exhibit the effect of the new cuts, we therefore choose the parameters M_1 and Y_1 such that leptogenesis takes place in the transitional regime from weak to strong washout. In this situation, sizable contributions to the final lepton asymmetry are generated for $z \sim 1$, such that the effects of the new cuts becomes relevant.

We choose for $N_{1,2}$ thermal initial conditions. In the early Universe, these may be established through interactions via heavy gauge bosons that freeze out at times before leptogenesis takes place. We assume that N_2 is maintained in equilibrium due to a Yukawa coupling with an additional lepton flavour, within which no asymmetry is produced. The coupling Y_2 is chosen smaller than Y_1 , such that washout effects from inverse decays of N_2 are negligible. (Explicitly, for the scenarios with $M_2 > M_1$, the largest error from washout through Y_2 occurs for $M_2/M_1 = 1.1$. Since the thermal suppression of N_2 compared to N_1 is very small in this case and we have chosen $Y_2/Y_1 = 1/2$, we expect that the washout is underestimated by 20%. The accuracy improves for larger ratios of $M_2 > M_1$ due to the thermal suppression of N_2 and its irrelevance for washout). While this appears as a somewhat special setup, we note that even when Y_2 is large, the vector of couplings of N_2 to the various left-handed lepton flavours defines a particular linear combination ℓ_2 of leptons that are washed out through inverse decays of N_2 . This linear combination can in general be linearly independent of the linear combination ℓ within which the lepton asymmetry through decays and inverse decays of N_1 is produced. The contribution to the asymmetry in ℓ that is orthogonal to ℓ_2 is then unaffected by the washout through N_2 [31]. (Note the different assignment of the heavy neutrinos to the indices 1,2 in that work). Therefore, the qualitative features of our particular setup should be relevant for parametrically more generic models of leptogenesis. However, it would still be interesting to study the effect of different initial conditions and the possibility of N_2 being out-of-equilibrium within future work.

As it is described in detail in Ref. [5], we obtain the numerical results as follows: First, we solve the evolution equations for $f_{N_1}(\mathbf{k})$ as a function of z . These, we feed into the washout term W and the source term S for the leptons, in order to solve for the lepton asymmetry in Eq. (1). The expansion of the Universe is taken into account when inserting the scale factor according to $M_{1,2} \rightarrow aM_{1,2}$.

The results for the lepton-number to entropy ratio Y_ℓ as defined in Ref. [5] are presented in Figure 3. Note that it is instructive to compare the particular panels with those in Figure 2. Again, we summarise some features for each of the particular values of M_2/M_1 :

- (A): For $M_2/M_1 = 1.1$, the hierarchical limit clearly underestimates the full result. This is anticipated, because the wave-function contribution to the full result is

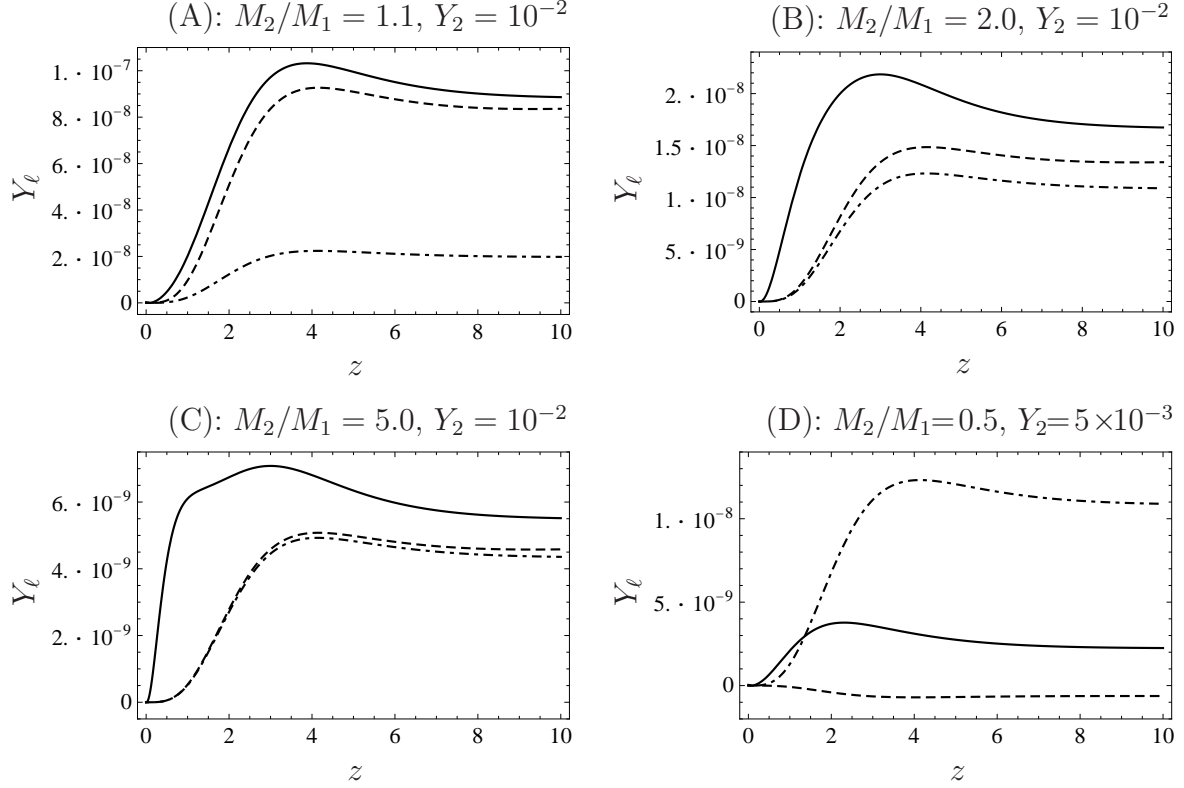


Figure 3: Evolution of the lepton asymmetry Y_ℓ over $z = M_1/T$. The choice of parameters is $M_1 = 10^{13}\text{GeV}$, $Y_1 = 2i \times 10^{-2}$. Solid: full result; dashed: result with contribution from cut through $\{\ell, \phi\}$ only; dot dashed: hierarchical limit $M_2 \gg M_1$.

resonantly enhanced for $M_1 \approx M_2$. The new cuts give rise to relative corrections at the 10% level. Note however that the absolute correction is larger when compared to scenarios (B) and (C). The relative correction is marginalised due to the resonant enhancement of S^{wf} .

- (B): For $M_2/M_1 = 2.0$, there are sizable deviations of the full result from the result with the cut through $\{\ell, \phi\}$ only, that arise in the region $z \sim 1$.
- (C): For $M_2/M_1 = 5.0$, the new contributions are of importance for smaller values of z when compared to scenarios (A) and (B). In the light of the thermal corrections that we anticipate to be important for small values of z , the quantitative result needs to be interpreted with care. The good agreement between the result from the standard cut through $\{\ell, \phi\}$ only and the hierarchical limit serves again as a consistency check for our numerical evaluations.
- (D): For $M_2/M_1 = 0.5$, we choose a smaller value for Y_2 , motivated by the requirement that the N_2 must not wash out the lepton asymmetry. The full result and the

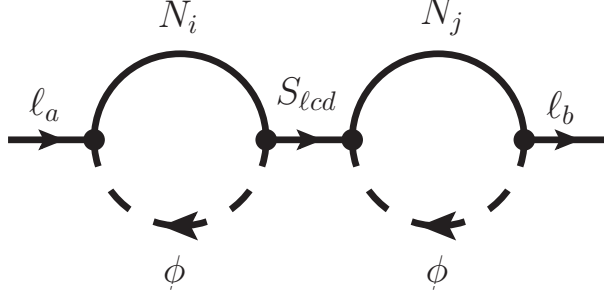


Figure 4: Wave function correction that contributes to the lepton asymmetry in flavoured leptogenesis.

result with the cut through $\{\ell, \phi\}$ only receive different contributions even for values of z of the order of a few, *cf.* Figure 2 (D). We expect therefore only a small contamination from theoretical uncertainties due to thermal corrections. Note that in this scenario most of the lepton asymmetry of the Universe originates from the new cuts.

4 Flavoured Leptogenesis

When we insert a loop of ϕ and N_2 as a wave-function correction into the propagator S_ℓ in the vacuum, no branch cut term due to on-shell ϕ and N_2 arises for kinematic reasons. Again, this holds no longer true in a finite temperature background. Therefore, CP -violating source terms can arise from the diagram in Figure 4. Since there is no lepton number violation, this does not yield a contribution to the asymmetry in models of unflavoured leptogenesis. However, this diagram may appear as a source in flavoured scenarios. In this Section, we give a rough estimate of this contribution, leading to the conclusion that it is generically negligible.

Within the CTP-formalism, the form of the Wightman type self-energy, that is given diagrammatically in Figure 4, reads

$$\begin{aligned}
 i\tilde{\Sigma}_{\ell ab}^{\text{wfl}\ell>}(k) = & -Y_{ia}^* Y_{ic} Y_{jd}^* Y_{jb} \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \\
 & \left\{ iS_{Ni}^>(p) i\Delta_\phi^<(p-k) iS_{\ell cd}^T(k) iS_{Nj}^T(q) i\Delta_\phi^T(q-k) \right. \\
 & -iS_{Ni}^>(p) i\Delta_\phi^<(p-k) iS_{\ell cd}^<(k) iS_{Nj}^>(q) i\Delta_\phi^<(q-k) \\
 & -iS_{Ni}^{\bar{T}}(p) i\Delta_\phi^{\bar{T}}(p-k) iS_{\ell cd}^>(k) iS_{Nj}^T(q) i\Delta_\phi^T(q-k) \\
 & \left. +iS_{Ni}^{\bar{T}}(p) i\Delta_\phi^{\bar{T}}(p-k) iS_{\ell cd}^{\bar{T}}(k) iS_{Nj}^>(q) i\Delta_\phi^<(q-k) \right\}.
 \end{aligned} \tag{23}$$

Compared to the unflavoured scenario, we have promoted the Yukawa couplings of the right-handed neutrinos N_i to a matrix Y_{ia} , where the first index refers to the right-handed neutrino and the second index to the left-handed lepton flavour. For the definitions of the model Lagrangian and the lepton propagator S_ℓ , we refer to Ref. [26].

Next, we insert $\cancel{\mathcal{F}}_{\ell ab}^{\text{wfl}\ell>}$ into the collision term (3). This again simplifies when substituting equilibrium propagators for N_2 , ℓ and ϕ and exploiting KMS relations. We furthermore choose to work in the flavour basis where the matrix of Standard Model lepton Yukawa couplings h_{ab} is diagonal. This is advantageous since off-diagonal components of $S_{\ell ab}$ are damped away quickly in this basis, provided the interactions mediated by h_{ab} are fast compared to the Hubble rate [14, 33–35], *cf.* Ref. [26] for a description within the CTP approach and for a numerical study of this effect. Note that this also implies that the off-diagonal components of the equilibrium propagator for ℓ are vanishing. We eventually obtain for the collision term

$$\begin{aligned} \mathcal{C}_{\ell aa}^{\text{wfl}\ell}(\mathbf{k}) = & - [Y_{1a}^* Y_{1c} Y_{2c}^* Y_{2a} - Y_{2a}^* Y_{2c} Y_{1c}^* Y_{1a}] \int \frac{dk^0}{2\pi} \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \\ & \frac{1}{2} \text{tr} \left\{ \left(iS_{\ell aa}^<(k) iS_{Ni}^>(p) i\Delta_\phi^<(p-k) - iS_{\ell aa}^>(k) iS_{Ni}^<(p) i\Delta_\phi^>(p-k) \right) i\text{Re} \left[\frac{1}{k^2 + i\varepsilon} \right] \right. \\ & \left. \times \left(iS_{Nj}^<(q) i\Delta_\phi^>(q-k) - iS_{Nj}^>(q) i\Delta_\phi^<(q-k) \right) \right\}, \end{aligned} \quad (24)$$

where ε is infinitesimal. In the zero-width limit, this term contains a divergence from the factor $1/(k^2 + i\varepsilon)$ that originates from the propagator $S_\ell^T(k)$, since $S_{\ell aa}^<(k)$ is proportional to $\delta(k^2)$. At finite temperature, this is regulated through the replacements

$$\frac{1}{k^2 + i\varepsilon} \rightarrow \frac{1}{k^2 - m_{\ell c}^2 + ik^0 \Gamma_\ell} \quad (25)$$

and

$$\delta(k^2) \rightarrow \frac{i}{2\pi} \left[\frac{1}{k^2 - m_{\ell a}^2 + ik^0 \Gamma_\ell} - \frac{1}{k^2 - m_{\ell a}^2 - ik^0 \Gamma_\ell} \right]. \quad (26)$$

Here, the thermal masses of the leptons are $m_{\ell a}^2 = h_{aa}^2 \zeta^{\text{fl},h}(k) + g^2 \zeta^{\text{bl},g}(k)$, where g is the $\text{SU}(2)_L$ gauge coupling. The functions $\zeta^{\text{fl},h}(k)$ and $\zeta^{\text{bl},g}(k)$ are of order T^2 for k of order T . They account for thermal mass corrections from flavour-sensitive and flavour blind interactions, and are discussed in more detail in Ref. [26]. Likewise $\Gamma_\ell = g^2 g^{\text{bl}}(k)$ is the finite-temperature width of the leptons, where $g^{\text{bl}}(k)$ is of order T when k of order T . When leptogenesis occurs at temperatures of roughly below 10^{11} to 10^{12} GeV but above 10^8 to 10^9 GeV , the τ -lepton Yukawa coupling h_τ is in equilibrium, while the electron e and muon μ couplings are yet out-of-equilibrium. In this situation, flavoured leptogenesis distinguishes effectively between two flavours, where flavour 1 is identified with τ and the coupling $h_{11} = h_\tau$ and flavour 2 with a linear combination of e and μ and with negligible Yukawa coupling $h_{22} \approx 0$. The integral (24) could possibly again be

evaluated numerically. However, we can estimate that from the factors (25) and (26), the k^0 -integration yields a factor

$$\frac{m_1^2 - m_2^2}{T^3 \Gamma_\ell^2} \sim \frac{1}{T^3} \frac{h_\tau^2}{g^4}, \quad (27)$$

for $|\mathbf{k}| \sim T$. The latter estimate follows from above assumption that only h_τ appears as a relevant Standard Model Yukawa coupling. Above suppression factor is to be compared to $1/T^3$ for a leptogenesis scenario with $M_1 \sim M_2$ but no pronounced resonant enhancement. Since $h_\tau^2/g^4 \ll 1$, we conclude that this contribution to flavoured leptogenesis is suppressed due to the large width of the leptons ℓ at finite temperature. In other words, the contribution to the CP -asymmetry is rendered ineffective because the separation between the resonances of the lepton quasi-particles is well within their overlap due to the finite widths.

5 Conclusions

In this paper, we have presented in the form of Eqs. (14) and (19) the first results for source terms that contribute to the lepton asymmetry in a finite-density background, but that are absent in the vacuum. In order for these to be relevant, we have seen that M_2 should not be much larger than T at the time of leptogenesis. The main features of the numerical evaluations are easily understood. First, M_2 should not be much larger than M_1 for the new effects to be sizable at larger values of z , *cf.* Figure 2. We reemphasise that the quantitative results for the scenario in Figure 3 (C) should be considered with great care since a substantial amount of the deviations incurred by the new effects is generated at very small values of z , where we expect thermal corrections to become relevant. Second, the effect is most pronounced in scenarios where a sizable amount of the asymmetry is produced for comparably small values of z , as it is the case for the transitional regime between weak and strong washout. As a loophole, we find that the largest effects arise within a somewhat unconventional scenario with $M_2 < M_1$, since in this situation the new cuts are important throughout the time of the out-of-equilibrium decays of N_1 . Therefore, relevant contributions from the new cuts also result for larger values of z . However, in regard of Ref. [31], where it is shown that decay asymmetries from the heavier singlet neutrinos generically survive subsequent washout, such a scenario may not only appear as a mere loophole. Provided the reheat temperature is large enough to produce the heavier singlet neutrino in the early Universe, the present work implies that a large contribution to the baryon asymmetry of the Universe generically results from the new cuts.

We mention that while this work has been in preparation, Ref. [36] appeared, where non-standard cuts are discussed as well. Since finite density effects are neglected, it is found there that the new cuts are only allowed within certain scattering processes. More precisely, in order to satisfy the kinematic thresholds for the presence of cut contributions in the vacuum, diagrams with additional radiation of Standard Model particles are considered. Note that if the resulting amplitudes were substituted as source terms into the

kinetic equations for the lepton asymmetry, they would be subject to the same Maxwell suppression that occurs also in the present case for $M_2 \gg T$. However, compared to the new source terms that we have derived in the present work, the terms discussed in Ref. [36] are subject to further suppression because of the insertion of additional coupling constants.

On the ground of the present results, it would be interesting to address the following points in the future:

- The results (14) and (19) should be generalised to include effects from deviations of the distribution f_{N_2} from equilibrium, which we expect to be non-zero in generic scenarios of leptogenesis.
- Systematic investigations of the parameter space for scenarios where the new cuts are relevant would be desirable. We expect the new cut contributions to be of crucial importance for phenomenological studies where it is assumed that $M_2 < M_1$, such as Refs. [30–32], where N_2 may or may not be in equilibrium. (Note the different definitions of $N_{1,2}$ in these works). Note that in case $M_2 \ll M_1$, thermal corrections may become important again, since the thermal masses of ϕ and ℓ at the time of the decay of N_1 may exceed M_2 . We expect the new cuts still to be important in such a situation, but the interpretations of the particular cut particles in terms of absorption processes may change to emissions and vice versa.
- In the present work, we have restricted ourselves to compute the lepton asymmetries for thermal initial conditions for $N_{1,2}$. This is in part motivated by the fact that for a vanishing initial density of N_1 , the final asymmetry is a remainder of an incomplete cancellation of a contribution that is created initially at small z , when N_1 is underabundant, and an opposite one through later decays when z is larger and N_1 overabundant. The fact that the cancellation is incomplete is because the opposite asymmetries are affected differently by the washout, since they occur at different temperatures. In order to obtain a quantitatively reliable result for the remaining asymmetry, a rather accurate prediction of the asymmetry that is created through inverse decays at small z is necessary. Due to the uncertainties because of thermal corrections, that we have emphasised in this paper, such an accurate prediction is presently not available. The situation somewhat improves for the thermal initial conditions that we consider within the present work, because N_1 is always overabundant and the produced asymmetry is therefore of the same sign for all values of z .

In regard of these points, we briefly comment on possible technical improvements that may prove very useful in order to increase the accuracy of the predictions for leptogenesis from the new cuts as well as from the standard cuts. It would be particularly interesting, if the following issues were addressed:

- As it should be clear from the discussions within this paper, the uncertainties due to thermal corrections for small z are problematic for the predictivity of the new

cut contributions as well as more generically for leptogenesis in the weak washout regime, in particular for vanishing initial conditions for N_1 . In order to resolve this issue, a calculation of the rates $N_1 \leftrightarrow \ell\phi$ for temperatures that are of order M_1 or larger is necessary. Both, the tree-level rates as well as the CP -violating loop effects need to be calculated. For that purpose, in particular the thermal masses and finite widths of ℓ and ϕ should be taken into account. First work into this direction has been reported in Ref. [37].

- In Ref. [25], sizable effects from the finite width of N_1 in the CP -violating source term have been reported. It needs to be verified, whether the initial conditions chosen in that work are applicable to the situation in the early Universe. Furthermore, the finite widths of ℓ and ϕ , which are much larger than for N_1 and have been neglected so far, need to be taken into account in a future calculation.

Considering these comments and a number of related papers, the present work may be regarded as a contribution to present efforts to improve the theoretical description of leptogenesis, to increase the accuracy of quantitative predictions and to extend their range of applicability.

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